Comment on Keren Wilson's "Aristotle on Continuity" (OPA 2015)

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Let me begin by saying that I am a complete amateur when it comes to Aristotle. In particular, I know almost nothing about his philosophy of mathematics. So I am not offering comments here as an expert, but only as a general reader. I shall raise two issues, but first some preliminaries.

I find Keren's paper interesting and thought-provoking. In particular, I like her careful attention to the nuanced meanings of the relevant terms employed by Aristotle. Her main aim is to resolve a difficulty, on behalf of Aristotle, concerning his distinction between contiguity and continuity. For Aristotle, continuity is a higher form of togetherness or unity than contiguity, since it has the extra requirement that the boundaries of the touching parts be actually one. Now this distinction, as Keren points out, seems to hold well in the case of sensible or material objects, but appears to collapse in the case of pure geometrical objects. And we cannot save this distinction, she further points out, by restricting it to material objects, because Aristotle seems to mean his distinction to apply to pure geometrical objects as well. That is the difficulty.

Keren proposes to resolve this difficulty by appealing to a central distinction in Aristotle, namely the distinction between actuality and potentiality. The strategy is to recast the distinction between contiguity and continuity in terms of actuality and potentiality and then show that the distinction, in the new terms, applies to pure geometrical objects as well as material objects. The recasting is as follows: continuity is linked to "actually one but potentially two [or more]", while contiguity is linked to "actually two [or more]".

This strategy seems to work well in showing that continuity, understood in terms of "actually one but potentially two [or more]", applies to pure geometrical objects as well as material objects. To use the examples Keren discusses: a geometrical line is, just like a shoe or a syllable or an animal body, an actually undivided unity, but at the same time it is also, again like these material objects, potentially divisible into two [or more] parts.

But it seems not very clear how *contiguity*, now understood in terms of "actually two [or more]", also applies to geometrical objects as well as material objects. Again, with material objects there seems to be no difficulty. But how are we to see that, e.g., when two lines touch each other, there are *actually two* points where they touch each other? Intuitively there seems to be just one point. It is true that we can get two points if we use the kind of argument Keren credits to Marco Panza, which uses the delimiting roles of boundaries as their criteria of identity. But there are objections to this kind of argument.

First, this Panza-style of argument relies on a tacit assumption that seems problematic. This is the assumption that the same geometrical object, e.g. a point, cannot perform different functions, e.g. delimit different lines, at the same time. But why should we assume this? This assumption seems quite counterintuitive. To see this, consider a different example from the one discussed by Keren. If, given a square, I draw a diagonal, *two* triangles will pop up. By a Panza-style argument I must have actually constructed two lines. But intuitively I have constructed just one, though it delimits two triangles at the same time.

One may say that for the above objection to tell against the Panza-style argument it must invoke the notion of geometrical *construction*. This may be so. But in any case, construction seems to be a central part of Greek geometry (many of the problems in the *Elements* involve construction or simply are construction problems) and it would seem that no one can ignore construction when interpreting Aristotle's philosophy of geometry.

Perhaps a user of the Panza-style argument can somehow manage to dispense with the notion of construction. But even so, support for the tacit assumption in question is still needed. It is here, it seems, that Keren's ascription, following Jonathan Lear, of a particular mathematical ontology to Aristotle comes in. She is tentative about, and does not elaborate on, how this ontology supports this kind of argument, but the basic idea seems to be as follows.

Geometrical objects (e.g. lines, points) are, according to this ontology, non-substantial particulars that inhere in or belong to the material bodies from which they have been abstracted (filtering out all the material properties). It is a consequence of this ontology that the geometrical figure of one material body is really distinct from the geometrical figure of another material body, because the figures are now conceived as particulars inseparably tethered to particular material bodies, rather like how shadows are tethered to what they are shadows of. If two material bodies are contiguous but not [relational] continuous with each other, then their geometrical figures, which are parasitic on them like shadows, will be really distinct from each other. The boundaries of the figures may coincide, but in that case there are still *actually two* coincident boundaries, because there are *two* distinct figures to be bounded. (To put the point more vividly: when two people shake hands, the shadows of their hands may, with a little contrivance, exactly coincide, but there are still actually *two* shadows at the same location, as becomes clear when their hands are withdrawn.)

This might give the Panza-style argument the needed support, and the ontology at issue may also have other virtues. But the price seems to be too high. For this ontology, by turning geometrical figures into non-substantial *particulars*, seems to lead to the massively unwelcome consequence of invalidating the practice of geometers in using *particular* figures to prove *universal* theorems: how can the results proved about *this* particular triangle on *this* particular sandbox or parchment be valid for all triangles?