A New Interpretation of Carnap's Logical Pluralism

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Abstract

Rudolf Carnap's logical pluralism is often held to be one in which corresponding connectives in different logics do not share a meaning. This paper presents an alternative view of Carnap's position, in which connectives can and do share a meaning in some (though not all) situations. This re-interprettaion depends crucially on extending Carnap's linguistic framework system to include meta-linguistic frameworks. I provide an example that show how this is possible, and give some textual evidence that Carnap could agree with this interpretation. Additionally, I show how this interpretation puts the Carnapian position much more in line with the position given in Stewart Shapiro (2014) than had been thought before.

1 Introduction

There is one slogan that many people who hold that Carnap was a logical pluralist agree upon: Carnap's pluralism is one in which a change in logic can occur only when there is a corresponding change in connective meaning. In this way, it is claimed that a Carnapian pluralism requires language change any time there is logical change. Logical change can only occur because of language change.

What I will show in this paper is that there is a different interpretation of Carnap (1950) and Carnap (1937), where he is better accounted for as a Shapiro (2014)-style pluralist. On this interpretation, if we extend what he says about linguistic frameworks to meta-linguistic frameworks, then when two linguistic frameworks are embedded in distinct meta-linguistic frameworks, the connective meanings in the two origina; frameworks may or may not mean the same thing. Logical change, then, can sometimes occur without language change. The argument for this proceeds in several steps. First, I will show that the question of whether two connectives in different languages share a meaning, when asked without a meta-linguistic framework in mind, has no answer on the Carnapian picture, and is meaningless. Second, I will suggest that the only way to make the question meaningful is to assess it with respect to a meta-linguistic framework. I give some evidence for thinking that Carnap would agree. Further, this will allow us to claim that, with respect to some meta-frameworks, connectives in different logics will share meanings, while with respect to others, they will not. This final step is what allows for logical change without language change. Finally, I show how this new interpretation puts the Carnapian position much more in line with that of Shapiro (2014).

2 The Traditional Carnapian View

It will be useful to explain carefully why the typical slogan about Carnap's view takes him to be claiming that change in logic requires a change in connective meanings.[1] In particular, the slogan readily makes sense of passages like the following:

[1]Author's who often seem to be making use of such a slogan include Allo (2015), Cook (2010), Hellman (2006), Restall (2002) and Russell (2013). Rarely does anyone assert that the slogan applies to Carnap's position, but much of what is said can be thought to imply it. In a way, this paper can be read as a cautionary note against taking the slogan seriously.

> Our attitude towards...requirements...[of a logic] is given a general formulation in the *Principle of Tolerance: It is not our business to set up prohibitions but to arrive at conventions.... In logic, there are no morals.* Everyone is at liberty to build up his own logic, i.e. his own language. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical [as opposed to scientific] arguments. (Carnap, 1937, p 51/2)
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> Let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols. (Carnap, 1937, p xv)

These passages are typically taken to entail two things. The first is that we ought to be tolerant of different logics, and as long as we can build a logic (which is given by its syntactic rules), and provide applications for it, that logic is legitimate. As long as we have two applications which call for different logics, this gets us the logical pluralism. The second is that the meanings of the logical connectives are given by their inference rules. In this way, it is very easy to (mistakenly) make the inference that changing the logic requires changing the connective meanings. If connectives are defined by their inference rules, and changing the logic just amounts to changing the rules, then changing the logic seem to just amount to changing the connectives.

What I will claim in the next two sections is that this "sloganed" position conflates two notions: that of building a language from rules, and that of building a language from rules which the builder knows to be a distinct from the rules of other languages. As I will show, Carnap meant the former, as the question which corresponds to the latter cannot be answered ("Do these two languages have connectives which mean the same thing?"). A builder can build any language she wants, as expressed in the "i.e., his own language" in the quote above. In this respect we have to be tolerant. However, a builder cannot claim that her language is different from any other without first making some other assumptions about a meta-framework in which she is making that claim.

3 Carnap's Position

As we saw above, Carnap says several things which make it reasonable to assume that any change in the rules of a logic is necessarily a change in connective meanings. In order to show this is not the case, we need details on three aspects of his view: we need to know what linguistic frameworks are, how they relate to connective meanings, and what a pseudo-question is.

Carnap claims a linguistic framework is "a system of... ways of speaking, subject to... rules" (Carnap, 1950, p 242). We will assume here that the "ways of speaking" Carnap mentions really are just the syntactic rules for a logical system.[2] Recall that the connectives are defined by "[letting] any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols" (Carnap, 1937, p xv). Thus, the meanings of the connectives just are the rules of the linguistic framework.[3]

To get at the notion of a pseudo-question, we need some preliminary definitions first. A theoretical question is one asked relative to a linguistic framework. It is internal to a linguistic framework, and asked assuming the rules of that framework. A non-theoretical question is one asked about reality itself, without a linguistic framework in mind. It is external to any given framework. There

[2]And possibly model-theoretic. Early quotes from Carnap suggest a purely proof-theoretic view of logic. Though he held that proof theory was the best way to "do" logic early in his career, his position about logic being a purely proof-theoretic endeavor changed after he met and spoke with Tarski, who convinced him that model-theory was a legitimate enterprise (see, for example, Carnap (1947)). This will, in effect, be further support for the position suggest here, as model theory clearly (or, at least more clearly than proof theory) requires a meta-theory.

[3]In particular, they will be given by the L-rules of the framework, which are the rules that govern transformations of logically true sentences into logically true sentences. We will not here concern ourselves with P-rules, which govern transformations of descriptively true sentences into descriptively true sentences. For more information, see (Carnap, 1937, p 133-5).

are two types of external questions: pragmatic questions and pseudo-questions. A pragmatic question is a question about which framework is best for a given purpose, and can be answered.[4] Pseudo-questions (non-pragmatic external questions) cannot be answered, on Carnap's view, and this is the sense in which they are illegitimate. For Carnap, a pseudo-question is "one disguised in the form of a theoretical question while in fact it is a non-theoretical [question]" (Carnap, 1950, p 245). Examples of pseudo-questions include questions of existence of abstract objects, which can only be answered relative to a given linguistic framework, and questions about the right logic, or which logical consequence relation is correct. Outside of a linguistic framework questions like this are only pseudo-questions.

Thus, linguistic frameworks are (for our purposes) syntactical logical systems, and their rules define the connectives. If we ask a question with respect to no linguistic framework, then it is external, and it is a pseudo-question unless it is pragmatic.

4 The External Question

We can then ask the following question: when do corresponding connectives in distinct linguistic frameworks mean the same thing? According to the slogan from section 2, the answer question on Carnap's behalf ought to be "never". A different, perhaps less drastic version of the slogan suggests the answer ought to be something like "connectives are defined by their rules so if we change the rules, we necessarily change the meanings of the connectives." In this way, we might say that if two linguistic frameworks share some connective meanings if they share some of the same logical rules. Each of these ways of answering the question

[4]Pragmatic questions are in principle answerable. There is some question as to whether they are actually external, though. See Steinberger (2015) for an interesting discussion about how pragmatically to select the appropriate linguistic framework.

("never", and "only when the rules are the same") are flawed. This is because the question of when two linguistic frameworks have corresponding connectives with the same meaning is unanswerable; it is an external question which is not pragmatic. It is not pragmatic, since it is not about framework selection, but it is external, because it is not asked with respect to some given linguistic framework. It is a pseudo-question. Even if two linguistic frameworks have postulates and rules which have exactly the same "shape" (i.e. are typographically the same) we still cannot ask the question, since we still have no postulated linguistic framework in which to compare the rules and postulates in question. The answer to the question, then, cannot be "never". The question is a pseudo-question, and so cannot be answered

What, though, if we postulated the existence of a meta-linguistic framework, one in which the rules and postulates of both object-frameworks played a role? This would, I will show, give us an opportunity to answer the question of when two distinct frameworks have the same logical terminology. Additionally, I will show that the answer to whether two object-frameworks have corresponding connectives with the same meanings in a meta-framework will vary depending on our pragmatic goals, and hence the meta-framework we select.

First, when both frameworks in question are considered from the vantage point of a meta-linguistic framework, we could answer the question "when do two linguistic frameworks have logical terminology which means the same thing?". The question is now being asked, not about what is *really* true, but about what is true with respect to the meta-framework. This makes it a theoretical question, and so it is no longer a non-pragmatic external question. So far so good. The new question is answerable, and not a pseudo-question.

Second, I hold that the same-meaning question will have different answers depending on our pragmatic goals, and hence meta-linguistic framework selection.

There will be no single meta-framework which will do the trick for us here, there might be a whole spectrum of them. As with object-level frameworks, we pick one meta-framework which suits our pragmatic aims and operate with it. Given this choice, and presumably given that each meta-linguistic framework will come equipped with some rules and postulates for determining when two terms have the same meaning, we can infer the following. With respect to some meta-framework embeddings, corresponding connectives will share a meaning. With respect to others, though, they will not be the same.

As an example, let us consider two meta-linguistic frameworks and two object level frameworks. One object level framework is classical and the other intuitionistic. The only thing that will concern us about the meta-linguistic frameworks are which rules they have for determining sameness of meaning. In this example, each is equipped with a different version of a double negation translation, and two terms are synonymous if they are inter-translatable.[5] The first meta-framework has the typical Gödel-Gentzen translation from classical to intuitionistic logic, call it T_1, and is defined inductively as follows:

1 if ϕ is atomic, then $T_1(\phi) = \neg\neg\phi$

2 $T_1(\phi \wedge \psi) = T_1(\phi) \wedge T_1(\psi)$

3 $T_1(\phi \vee \psi) = \neg(\neg T_1(\phi) \wedge \neg T_1(\psi))$

4 $T_1(\phi \rightarrow \psi) = T_1(\phi) \rightarrow T_1(\psi)$

5 $T_1(\neg\phi) = \neg T_1(\phi)$

It is a known result that ϕ is provable classically if and only if $T_1(\phi)$ is provable constructively. In this sense, we have a translation between classical and intuitionistic logic. Now, consider a second translation, call it T_2, and assume that this is the translation available in the second meta-framework. T_2 is the same as

[5]Thanks to ?? for suggesting this example.

T_1 for all clauses expect conjunction. The conjunction clause for T_2 is

2* $T_2(\phi \wedge \psi) = \neg\neg(T_2(\phi) \wedge T_2(\psi))$

T_2 also has the desired property that ϕ is provable classically if and only if $T_2(\phi)$ is provable constructively (see appendix for proof). Here, though, we can ask the following question: do intuitionists and classicists mean the same thing by "$\wedge$"? Well, if the translations provide us with a relation of synonymy, then because conjunction is translated as conjunction via T_1, while it is translated as a double negation of conjunction by T_2, we have sameness of meaning via T_1, but difference in meaning via T_2. In essence, it seems that if we are using T_1 they do, while if we are using T_2 they do not. Depending on our theoretical purposes, or on which meta-framework we are using, we will have access to different translations, and different logical terms will be synonymous.

On the face of it, it might seem unlikely that we will be able to use different translations depending on our purposes and aims. However, consider the difference between translating between intuitionism and classicism for the purposes of a logic class, where the Gödel-Gentzen might be sufficient and simplest, and translating between the two for the purposes of writing computer programs, where the Kolmogorov translation might be more successful (the Kolmogorov translation of ϕ is generated by affixing a double negation to every subformula of ϕ). Here, depending on our purposes, one translation will be better than another, and so we ensure that we select a meta-framework which can make sense of that translation, and thereby may generate differences in the "same meaning" relation.

5 Carnap's Agreement

There is an argument to be made that Carnap would agree with the interpretation from section 4. I will point to three factors here as evidence.

First, let us reconsider the quotes of Carnap given above to support the traditional view.

> Our attitude towards...requirements...[of a logic] is given a general formulation in the *Principle of Tolerance: It is not our business to set up prohibitions but to arrive at conventions.... In logic, there are no morals.* Everyone is at liberty to build up his own logic, i.e. his own language. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical [as opposed to scientific] arguments. (Carnap, 1937, p 51/2)
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> Let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols. (Carnap, 1937, p xv)

Here, it is important to note that we never explicitly have the claim that we can tell when two languages are distinct. Everyone might still be at liberty to build up their own language, but nothing here explicitly prevents that in order to compare any two languages, we must first embed them in a meta-language. In fact, given that whether two terms share a meaning is a theoretical question, we are *required* to address meaning questions from within a linguistic framework, and so are required to embed any framework into a meta-framework to answer the question.

There is some sense, when interpreted as speaking loosely, that Carnap may be thought to be saying exactly the opposite here. On the face of it, what he is saying here is something like "we recognize rules and postulates without needing any theoretical apparatus, and can tell when they are the same just by looking at them". However, I think this is reading the Carnapian position too loosely. First, the quote occurs in an introduction, so we can expect it to not be as rigorous as we would have hoped. Second, in order to interpret Carnap consistently (which we want to do), we ought to interpret him as not putting forward this loose interpretation as a serious component of his view. As I suggested above, meaning questions are theoretical questions on his view, and thus must be asked with respect to some framework to be answered. Thus, I claim, the loose interpretation just will not do here.

Taken as not speaking loosely, what Carnap can be thought to have said in these passages is that, though the rules do determine a connective's meaning, they only determine a meaning *within a framework*. Without considering the two frameworks from the perspective of a meta-framework, there is no way to know whether the rules which determine the meanings of the connectives are the same. There are two reasons we are prevented from knowing this: first, because sometimes rules with the same shape will wind up meaning different things under the embedding into a meta-linguistic framework, and second because sometimes rules which intuitively look different will end up meanings the same thing under the embedding into a meta-linguistic framework.[6] The passage above, then, can only be referring to rules determining a meaning for the connectives *within a given framework*. The passage makes no claim about considerations from outside of a framework (nor should it).

Second, it seems as though Carnap explicitly agrees that language comparison must be done in a meta-language (here, languages just are linguistic frameworks).

[6]As an example of the first case, consider two linguistic frameworks, each defined by the typical left and right logical rules for a sequent calculus, but one which has the full complement of structural rules, and one which does not have weakening. The connectives in both are defined by their logical rules. In this case, when we embed these two frameworks into a meta-framework, even though the rules look the same, it might not be best to translate them as the same rules. In effect, the logics will be different (the first classical, the second relevant), and so if we follow through on the traditional Carnapian slogan, the connective meanings ought to be different. This is an example where rules which look the same may not be the same once embedded into a meta-framework.

As an example of the second case, consider an adaptation of a case from Shapiro (2014). Here, we have two mathematicians, one classical and one constructive, discussing some form of analysis. The classicist normally defines her connectives via truth conditions, and the constructive mathematician normally uses proof conditions. However, during their exchange, and for the purpose at hand, they never discuss the connectives, nor do they consider any results which are not acceptable to both of them. In this situation, it makes sense to talk about them as though they are "speaking the same language", that is, as though they are both using connectives which mean the same thing. This is an example where rules which do not look the same may in fact be the same once embedded into a meta-framework.

Consider the following from (Carnap, 1937, section 62). There, he discusses translations from one language into another. These translations must occur within a meta-language:

> The interpretation of the expressions of a language S_1 is thus given by means of a *translation* into a language S_2, the statement of the translation being effected in a syntax-language S_3... (Carnap, 1937, p 228)

If all translations are effected in a meta-language, then there is reason to suspect that when using two distinct meta-languages, sometimes a term in S_1 will be translated into a particular term in S_2 and sometimes to a different term in S_2.

Third, Carnap holds that just what a translation must do is dependent on our theoretical goals. If the above is correct, then it matters what S_3 is. However, if there were only one type of translation, then we might expect that the relationship between S_1 and S_2 to always look the same no matter what we select as S_3. Carnap himself suggests that what a translation is required to do is variable; depending on our theoretical aims, a successful translation will be different. Sometimes it "must depend upon a reversible transformance, or it must be equipollent in respect of a particular language, and so on" (Carnap, 1937, p 228). But it need not always be as such. It might not be reversible, and it might not be equipollent (bijective). Depending on what we are wanting our translation to do, we impose different conditions on it being good. So what a translation must do, and what it must preserve is a pragmatic choice.

Thus, given that the original quotes taken to support the traditional view can be reinterpreted, and that Carnap explicitly says that two languages must be discussed with respect to a meta-language, and what translation have to preserve is variable, it looks like he will support the interpretation given in section 4.

6 The Shapiro Position

Finally, I would like to suggest that the section 4 interpretation puts Carnap in a position much closer to that of Stewart Shapiro's than one might have thought. I will look at two scenarios from Shapiro (2014).[7] Shapiro claims that

> For some purposes...it makes sense to say that the classical connectives and quantifiers have different meanings that their counterparts in intuitionistic, paraconsistent, quantum, etc. systems. In other situations, it makes sense to say that the meaning of the logical terminology is the same in the different systems. (Shapiro, 2014, p 127)

The purposes/situations in question here can be thought of as something like the S_3 discussed above. Shapiro's two examples come from comparing classical analysis and smooth infinitesimal analysis,[8] and asking whether the logical systems required for each have connectives which mean the same thing or something different. In the first scenario, we compare the systems when we are interested in differences between the logics themselves. In the second scenario, we compare the systems in terms of their mathematical consequences. For example, in the first case, two people might be comparing axioms of each system; they might discuss whether the existence of nilsquares is possible. In the second, they might be comparing whether both systems prove some theorem; they might discuss whether both system prove the intermediate value theorem.

[7] See also ??

[8] Smooth infinitesimal analysis (SIA) is an intuitionistic analysis system, in which all functions are smooth. Importantly, it is such that 0 is not the only nilsquare (elements whose square is zero, i.e. elements x such that $x^2 = 0$). This is because every function is linear on the nilsquares. From this, it follows that 0 is not the only nilsquare even though there are no nilsquares distinct from 0. This would be inconsistent in classical logic (because of the validity of LEM), and so intuitionistic logic is required. More formally, in a classical system, the sentence $\neg\forall x(x = 0 \vee \neg(x = 0))$ is a contradiction. In an intuitionistic logic, since the law of excluded middle is not valid, the two sentences can both be true. Importantly for us, the SIA system has a very simple and straightforward proof of the fundamental theorem of the calculus (that the area under a curve corresponds to its derivative). Rather than, as usual, taking approximations of the rectangles under a curve as they approach a width of 0, we take a rectangle under the curve which has the width of a nilsquare. No approximations are necessary, and we do not need the concept of "approaching zero". See Bell (1998) for more details.

Here is the rub: Shapiro claims that in the first case, "[it is] natural to speak of meaning shift" (p 128), while in the second case, "it is more natural to take the logical terminology in the different theories to have the same meaning" (p 130). In our language, in the first case, the logical connectives are different in each system, while in the second they are the same. To put this in Carnapian terms, let classical analysis and smooth infinitesimal analysis be the first two linguistic frameworks, or S_1 and S_2. Then, when we are discussing the logics of the systems, we will find ourselves in a meta-linguistic framework where the only translations between S_1 and S_2 will be non-trivial and complex. In the second case, we ought to find ourselves in a meta-linguistic framework where we will be able to produce a trivial translation between CA and SIA. I suspect in the second case, the translation given by the map which identifies homophonic terms will be available. In the first case, however, because our meta-language is so interested in the details of each system, such a homophonic translation will not be robust enough, and we will need something more like a Gödel-Gentzen double negation translation, or a translation which uses the modal logic $S4$. This accords very well with they way Carnap seems think about translation and cross-framework meanings as discussed in section 5.

This means that Carnap may very well be a Shapiro-style logical pluralist (especially given what he says about what a translation needs to preserve above), which would make the slogan above mistaken.

7 Conclusion

It is often claimed that Carnap must hold that there is language change whenever there is logical change. However, this assumes that we can ask questions about meanings outside of any linguistic framework. This assumes that we can answer the question "does logical change require language change *really*?". This is illegitimate on Carnap's view. What we should be asking is "does logical change require language change in framework X?". By doing so, we embed the external question into a linguistic framework, and thus make it answerable. Additionally, when we embed the question into an additional linguistic framework, we see that sometimes there is meanings change when there is logical change, and sometimes there is not. In a sense, this means that Carnap must be Carnapian about the meta-theory.

8 Appendix

The following is a proof that ϕ is provable classically if and only if $T_2(\phi)$ is provable constructively. Recall that T_2 is defined inductively as follows

1* if ϕ is atomic, then $T_2(\phi) = \neg\neg\phi$

2* $T_2(\phi \wedge \psi) = \neg\neg(T_2(\phi) \wedge T_2(\psi))$

3* $T_2(\phi \vee \psi) = \neg(\neg T_2(\phi) \wedge \neg T_2(\psi))$

4* $T_2(\phi \to \psi) = T_2(\phi) \to T_2(\psi)$

5* $T_2(\neg\phi) = \neg T_2(\phi)$

To prove this we need to show the following

Theorem 1. *1. ϕ is equivalent to $T_2(\phi)$ classically*

2. If ϕ is provable classically, then $T_2(\phi)$ is provable constructively

Proof. Since we have already assumed that these results hold for the Gödel-Gentzen, and the proof is done inductively, we need only focus on proving the inductive cases for the conjunction clause **2***.

1. To show $\phi \wedge \psi$ is equivalent to $T_2(\phi \wedge \psi)$ classically, we start by assuming the inductive hypothesis that ϕ is equivalent to $T_2(\phi)$ classically, and ψ is

equivalent to $T_2(\psi)$ classically. Then we have $T_2(\phi \wedge \psi) = \neg\neg(T_2(\phi) \wedge T_2(\psi)$. By the inductive hypothesis, $\neg\neg(T_2(\phi) \wedge T_2(\psi)$ is equivalent to $\neg\neg\phi \wedge \psi$ classically, which is equivalent to $\phi \wedge \psi$ because of double negation elimination.

2. To show that if $\phi \wedge \psi$ is provable classically, then $T_2(\phi \wedge \psi)$ is provable constructively, we again assume the inductive hypothesis that if ϕ and ψ are provable classically, then $T_2(\phi)$ and $T_2(\psi)$ are provable constructively. Now, if $\phi \wedge \psi$ is provable classically, then ϕ and ψ are provable classically. By the inductive hypothesis, $T_2(\phi)$ and $T_2(\psi)$ are provable constructively. But since $\wedge$-introduction and double negation introduction are valid constructively, we have that $\neg\neg(T_2(\phi) \wedge T_2(\psi)$ is provable constructively. Thus, $T_2(\phi \wedge \psi)$ is provable constructively as desired.

□

Corollary 1.1. *ϕ is provable classically if and only if $T_2(\phi)$ is provable constructively*

Proof. The forward direction of the biconditional is just 2. The reverse direction follows from 1 and the fact that if a theorem is provable constructively, the it is provable classically. □

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