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Comment on Mitchell-Yellin

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In general, intuitionist logicians cannot accept the inference: ?("x) Fx ├ (∃x) ?Fx. This is because the proof of (∃x) ?Fx from ?("x) Fx requires a proof by contradiction that relies upon two uses of reductio ad absurdum. In both cases, the intuitionist can derive the contradictions; however, the next and final move is to drop the double negation in order to arrive at (∃x) ?Fx and this is exactly what intuitionist logicians are unwilling to do. On one hand, Benjamin Mitchell-Yellin?s paper is an attempt to show that the intuitionist should accept the inference, but *only in those instances in which the set ?x? that is being quantified over is a* *<u>decidable</u> set*. The notion of decidability is important because it allows for the possibility that an arbitrary member of *x* can be picked out so as to satisfy the existential quantifier. If the set is not decidable, the intuitionist logician has no reason to accept the quantifier exchange. Mitchell-Yellin?s example meets this requirement and is, therefore, a demonstration of a scenario in which an intuitionist would be justified in accepting the quantifier exchange; however, this qualification is neither new to intuitionists nor do they reject its import.

For this reason, I wish to turn the focus to what I take to be the second purpose of Mitchell-Yellin?s paper. That is, the issue about what qualifies as decidability for set. In the example, Dave obviously thinks that, at least in this case, it is acceptable to use the quantifier exchange. His boss does not agree. Since the boss is an intuitionist, he must not think that the set of boxes in the warehouse is decidable?at least not as long as Dave obeys his orders to not go inside the warehouse. For the sake of drawing a clearer distinction between the decidability and indecidability of the set of boxes in the warehouse, let us assume that Dave is also an intuitionist. Since Dave is willing to accept the quantifier exchange in this context, Dave must think that the set of boxes is a decidable set.

It is now possible to focus on the criteria for the decidability of a set. Mitchell-Yellin has introduced two of Michael Dummett?s theses that define Dummett?s justificationist position on verification. I summarize these two points in the following way:

(i) In principle, statements about observable states of affairs can be known to be true.

(ii) A statement is true if there is a direct justification of it and an indirect justification of a statement shows that a direct justification exists for it.

For our current purposes, the more important of these is (i). If an intuitionist accepts (i), he should also accept the claim that the set of boxes is decidable; therefore, he should accept the quantifier exchange in this case. Thus, Dave must accept (i), or something to the same effect, while his boss denies (i).

If a person finds the boss?s position highly problematic to the extent that it creates a sort of ?practical contradiction??although certainly not a logical contradiction?there are two viable ways for that person to resolve this issue. First, intuitionistic logic can be rejected. Second, the negation of (i) can be rejected. These options simply follow from the fact that, in a proof by contradiction, any of the assumptions that lead up to the contradiction can be taken to be false in order to resolve the contradiction. Usually, however, it is clear which of the premises the person wants to reject. Yet, in this case, either assumption [i.e. intuitionistic logic is correct OR (i) is incorrect] is fair game. This choice becomes even more problematic if it is determined that the sets that these two assumptions belong to are themselves indecidable. Also, it is interesting to note that, if intuitionists choose to reject the claim that (i) is false, the resulting claim is the double negation of (i). In this case, since intuitionists will not allow the removal of the double negation, the end result is ? ? (i) and not simply (i). Thus, the intuitionist need not accept (i) but only ? ? (i), which, according to intuitionists, need not be exactly the same as (i).

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