Abstract

Soritical predicates, such as 'bald', 'child', 'tadpole', 'heap', and so on, force **prima facie** compelling paradoxes upon defenders of principles of classical logic, e.g., the law of bivalence. But the burden of showing that defenders of classical logic are committed to paradoxes lies on those that propound them. I invent such a propounder, Sir Sorities, to emphasize where the dialectical burden lies, and I argue that we should reject the premises of a sorites argument. For there is an illicit move from our linguistic mastery of natural language predicates such as 'bald' to a logical predicate 'F' standing for a unique and logically simple property. Our communicative mastery of 'bald' rather suggests that soritical predicates are logically complex, and we should demand some analysis of the soritical predicate before buying into sorites premises.

1.0. Introduction

I aim to convince you that you do not quite know what bald means. This applies equally to a number of other words: heap, pile, child, tadpole, chair, and so forth. Such words and then some are used to present various versions of an old philosophical puzzle: the sorites.

Sorites puzzles are usually presented as a paradox, or at least as a puzzle that requires a solution. Galen took the sorites seriously enough to deny that many ordinary objects exist:

> According to what is demanded by the analogy, there must not be such a thing in the world as a heap of grain, a mass or satiety, neither mountain nor strong love, nor a row, nor strong wind, nor anything else which is known from its name and idea to have a measure of extent or multitude, such as the wave, the open sea, a flock of sheep and herd of cattle, the nation and the crowd. [5, 58]

Sorites puzzles, then, are at least puzzles. Rosanna Keefe and Peter Smith write, “Arguments with a sorites structure are not mere curiosities”, pointing out that sorites reasoning force one to accept the ethically untoward conclusions of “slippery slope” arguments [8, 3-4]. Keefe elsewhere writes of the sorites, “...the premises are highly plausible, the inference seems valid, but the conclusions are absurd.” [9, 1] Timothy Williamson views the sorites as a challenge to “the core of classical logic”, namely, “the principle of bivalence” [15, 1]. We have some pressure to answer the challenge they pose. I attempt some answering below.
2.0. Sorites Arguments

The following are typical characterizations of sorites paradoxes:

[A sorites is like this]: It is not the case that two are few and three are not also. It is not the case that these are and four are not also (and so on up to ten thousand). But two are few: therefore ten thousand are also. [5, 58]

Base Step: A one day year old human being is a child.
Induction Step: If an \( n \) day old human being is a child, then that human being is also a child when it is \( n + 1 \) days old.
Conclusion: Therefore, a 36,500 day old human being is a child. [13, §3]

[These] sorites [follow] the inference pattern known as mathematical induction:

\[
\begin{align*}
F_{a_1} \\
\forall n(Fa_n \rightarrow Fa_{n+1}) \\
\forall n(Fa_n)
\end{align*}
\]

These are paradigmatic sorites puzzles. But let me pose more strongly the soritical challenge:

Imagine that a stranger accosts you, introducing himself as ‘Sir Sorites’, and exclaims excitedly, ‘I heard through the grapevine that you accept classical logic, including that most horrid law of bivalence!’ You retort, as indignantly as a civilized tone permits, that classical logic has yet to lead you astray, and that you feel you have compelling reasons to let classical logic be your guide. Sir Sorites, hearing this, cackles maniacally, after which he leans close (too close for comfort, really), and whispers, ‘But have you a solution to the dreaded sorites? For they force a paradox upon you, and hence, a pox on adherents of classical logic!’

That stops you in your tracks, because you have a philosophical bent; and when someone tells claims to have a defeating argument for your dearest beliefs (as is the belief in classical logic, at least to many), then you wish to hear the indictment, and so as to answer the charge or abandon the erroneous opinion, as the case might be. So you discard your impatience

\(^1\) \( n \) will range over natural numbers, i.e. the positive whole numbers (\( \mathbb{N} = \{1, 2, \ldots\} \)), and \( k \) over \( \mathbb{N} \cup \{0\} \).

\(^2\) The Line-Drawing Sorites and the Conditional Sorites are similar.
with Sir Sorites, and ask for his supposed paradox, bracing for the blow.

The challenge for us is how to reply to such an interlocutor. The point of the foregoing is to arrange a dialectical situation in which the sorites puzzle is presented. For, having never heard a sorites puzzle before (let us pretend), the goal of your interlocutor is to force you to accept the paradoxical conclusion. We shall dissolve the puzzle, so to speak, if we can meet the dialectical challenges Sir Sorites raises. Now let us see how Sir Sorites proceeds.

*3·0. From Linguistic Uses to Logical Premises

Sir Sorites begins, naturally enough, at the beginning. ‘You agree,’ he sneers, ‘that some men are bald, say, those without a hair above the eyebrows!’ (Sir Sorites understands that proper dialectical procedure requires that his opponent accept the premises he proposes before continuing.) This is just $F_{a_1}$ above. We can adjust the case to fit the myriad soritical arguments: some humans are children (e.g., a one-month-old human is a child), some collections of sand are not heaps (e.g., one grain of sand is not a heap), and so on.

Let us suppose, for now at least, that we accept $F_{a_1}$, whatever soritical predicate ‘$F$’ we specify. This amounts to taking seriously, from the logical point of view, our linguistic use of ‘bald’ (or whatever our ‘$F$’ is) to specify some, but not all, things in a collection. Now not every linguistic use has a logical analogue; exclamations do not, for example. And linguistic uses of predicates may have seemingly bizarre logical analogues, e.g., as in Bertrand Russell’s analysis of ‘The present King of France is bald’ [10, 884]. This partly depends on our procedure of logical analysis and upon our logic of choice. So not every linguistic predicate has an analogue in some single logical predicate that stands for a unique atomic property; soritical predicates, say, like ‘bald’, may be shorthand for a very long disjunction. There is no royal road from linguistic predicates, e.g., predicates of English, to logical predicates, i.e., predicates in our logical vocabulary, which represents an atomic (unanalyzed) property.
But we must accept that we successfully use soritical predicates in communicating (e.g., we describe others as bald and our hearer understands which person we thereby pick out), and we hold beliefs whose propositional content includes some analogue (perhaps a very long disjunctive property) of a soritical predicate (e.g., we believe that some people are bald). We thus seem forced to accept that $F a_1$. Sir Sorites wins this round.

But notice that we can challenge the translation of the English predicate ‘bald’ into the logical predicate ‘$F$’. Recall that Sir Sorites emphasized our agreement that we in practice describe people using ‘bald’; there is, however, quite a leap in moving from this practical employment of ‘bald’ to a logical representation of ‘bald’ as a simple (unanalyzable) property. In fact, our success in using ‘bald’ to specify people to others suggests rather that ‘bald’ is very much analyzable; individuals with no hair, or large bald spots, or a toupee, may be equally specified using ‘bald’. This rather suggests that we should reject ‘$F a_1$’ as a preposterous candidate for a logical representation of our linguistic practice; rather, we ought to accept some schematic formula ‘$\phi$’ as a temporary stand-in for ‘bald’ in our logic, and $\phi a_1$, leaving open the exact analysis (and semantics) of ‘bald’ in our logical vocabulary.

The distinction between our practical goals and our logical goals is crucial. For Sir Sorites cannot rely on purely logical considerations to foist upon you the premises of a sorites puzzle. Sir Sorites appeals to other considerations, say, what you would typically be disposed to report as your belief. Sir Sorites is not basing his argument on propositions of Principia Mathematica alone. So if Sir Sorites appeals to some pragmatic datum to establish a logical point, we should scrutinize the appeal very carefully before we decide to accept this premise. And Sir Sorites’ inference, which purportedly justifies his demand that we accept $F a_1$, from a linguistic predicate to a logical one is, upon scrutiny, unduly hasty.

Suppose, though, for the sake of argument, that we accept $F a_1$, thereby admitting ‘$F$’ into our logical vocabulary. Let us see where Sir Sorites’ argument moves next.
4.0. What A Difference A Day Makes

Now Sir Sorites has been grinning (maliciously, or is it just the lighting?) all the while, and rocking backwards and forwards on his feet (again, close enough to you to be impolite). He now urges, ‘You agree, too, that a bald man, whatever characterizes his baldness, and however many hairs he has already, remains bald if you add one hair to his head?’ He lowers his voice, commanding almost inaudibly, ‘Yes, accept it! You are so, so close to paradox!’

But you may, in all fairness, find this most recent premise rather bewildering. Formally-speaking, the logical analogue looks something like this: $\forall n(Fa_n \rightarrow Fa_{n+1})$. But does your successful linguistic use of ‘bald’ tell you whether to adopt this one way or another? Try ‘heap’ on for size, too. And how about ‘child’ - does the word ‘child’, to be successfully used, pick something that always remains invariant from one day to the next day?

Consider the word ‘child’. There is a legal sense of this word on which a day makes an enormous difference; you switch from being a legal child to a legal adult in a single day. This use of ‘child’ appears to pick out a unique and genuine property, namely, the amount of time (with the Earth as a reference frame) a person has lived. On this understanding of the predicate ‘child’, we should deny the second premise of the sorites argument outright.

Suppose, then, Sir Sorites insists we stop acting like children, and appeals to other uses of child - say, that used to refer to a person’s maturity. But suppose a person we would describe as a child watches *Mr. Smith Goes to Washington*. Perhaps we shall find awoken in her, call her ‘Clarissa’, a profound sense of public duty and moral responsibility. Whereas, the day before Clarissa watched *Mr. Smith*, she neglected her chores and cared not for the citizenry writ large, she now wishes to form a public policy debate club at her high school, to volunteer her time to political campaigns, and to inform others regarding legislation before her state and national legislature. Clarissa matured, we might say, in less than a day, such that we would now describe her as an adult, despite her being a ‘child’ in the legal sense.
The upshot of these considerations appears to be that soritical predicates, like ‘bald’ and ‘child’, are logically complex, not logically simple; the inference from the English predicate to a logical one is in general illicit, and we have reason to be suspicious of the move for nearly all the soritical predicates. Rather, they appear *prima facie* to be shorthand for a complicated disjunction, or a disjunctive schema or description, whose representation as ‘\( F \)’ is just misleading. So which ‘disjunct’ of ‘child’ might we mean? There are many we might choose. But Sir Sorites has yet to specify which logical predicate he means as the analogue of ‘child’, and above are two potentially logical predicates on which we have ample reason to reject the second premise of the sorites (as well as the first premise).

Let us summarize the morals of the foregoing discussion of ‘child’. First, we could select (or assign) similarly definite senses of ‘bald’, ‘heap’, ‘chair’, and so forth, and by analogous reasoning find ourselves rejecting Sir Sorites’ second premise. Second, there is not obviously a straight road from our successful linguistic use of soritical predicates to logical acceptance of the second premise of a sorites argument, for the semantics of a soritical predicate is not abundantly clear solely in virtue of our linguistic mastery of that soritical predicate’s uses. Finally, soritical predicates appear to pick out bundles of features of the world to achieve successful communication. ‘Child’ might be used to direct your attention to the shortest person on the playground; but if a bearded dwarf is present, you might instead look for a beardless person, and if a female dwarf is present, you might instead look for the person eating sand. But you might instead of have used the word to refer to your roommate that could not cook, or the friend that throws tantrums in public. The list of atomic (logically representable as ‘\( F \)’ properties we might mean by ‘child’, ‘bald’, and so on, could go on.

The premises of a sorites argument are advanced on the basis of your so-called ‘understanding of what ‘chair’ (say) means’. This can only refer to one’s linguistic mastery of such predicates. But this mastery does not obviously transfer logically, meaning, we cannot read off our use of linguistic predicates like ‘bald’ the logical semantics for such terms; nor can
we be forced to admit such terms into our logical vocabulary. For there is reason to suppose that such soritical predicates are logically complex, not simple or unanalyzable, especially in light of our capacity to use such predicates in a variety of situations. Let Sir Sorites provide an analysis of ‘bald’ or ‘child’ into terms that we already consider primitively understood, or terms that we previously accepted into our logical vocabulary, and then we shall be in position to judge whether his premises should be affirmed or denied. And our agnosticism as to the logical representation of ‘bald’ need not force us to stop using ‘bald’ in daily life, if we believe that the distinction between the logical predicate and the linguistic predicate is a sound one. For if we cannot be forced to accept a logical predicate in virtue of our linguistic use of a soritical predicate, then we cannot be forced to abandon our use of a linguistic predicate in light of our rejection of the logical predicate.

*5.0. A Rejection, Not a Denial

One might object that my proposal amounts to denying the inductive premise in the classical sorites argument, i.e., to negating $\forall k \in \mathbb{N}[(k \in A) \rightarrow (k + 1 \in A)]$. But, by some uncontroversial inference rules of quantificational logic, this implies $\exists k[(k \in A) \& (k + 1 \notin A)]$, i.e., that there is a sharp dividing line between ‘is bald’ and ‘is not bald’ such that adding a single hair transforms a bald person into a non-bald person. Yet this is felt, quite strongly, to be quite wrong. The worry is a frequent concern in the literature on vagueness:

Any “normal” observer...finds extreme difficulty in “drawing the line” between chair and not-chair. Indeed the demand to perform this operation is felt to be inappropriate in principle: “chair is not the kind of word which admits of this sharp distinction” is the kind of reply which is made... [1, 433]

$^3 \forall k \in \mathbb{N}[(k \in A) \rightarrow (k + 1 \in A)]$ means, ‘For all $k$ in $\mathbb{N}$, if $k$ is an element of $A$, then $k + 1$ is an element of $A.$’ $\exists k[(k \in A) \& (k + 1 \notin A)]$ means, ‘There is a $k$ in $\mathbb{N}$ such that $k$ is an element of $A$ and $k + 1$ is not an element of $A.$’ The reasoning that leads to this conclusion is also called the “Line-drawing Sorites” [7, §3].
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[We might try to] reject the induction step of the argument. The initial appeal of this move is dampened by the recognition that rejecting the induction step is tantamount to asserting its negation. For the negation of the induction step is equivalent to the proposition that there is a precise minimum number of grains of sand necessary for being a heap. In other words, there must be a sharp division point between heaps and non-heaps. [14, 55]

James Cargile phrases this worry rather differently; his soritical predicate is ‘tadpole’:

It seems clear that [being a tadpole] is correctly ascribable to [the tadpole after] 1 [second] and [being a tadpole] is not correctly ascribable to [the tadpole after] 43,545,600 [seconds]...However, from [these two apparently unquestionable facts], it is easy to derive, using...the least number principle, C. (∃n)(P(n)& ∼ P(n+1)). [T]he least number principle is that if the number 1 has a certain property and a larger number \( n \) does not, then there is a least number among the set of numbers between 1 and \( n \) which do not have the property... [3, 193, my formatting]

The worry is that rejecting the second premise requires acceptance of its negation, which forces the imposition of a sharp separation of ‘bald’ and ‘non-bald’ that no ordinary English-speaker would countenance, and similarly for other soritical predicates.

Focusing on how Cargile poses the problem, we note that he relies on the applicability of the least number principle to stages of a tadpole’s development [16, 269]. But we lack justification for applying the least number principle to stages of a tadpole’s development, which do not, absent an analysis of ‘tadpole’, obviously share the formal properties that warrant applying the least number principle [4, 338-339]. That is, until the propounder of soritical paradoxes puts a single definite property representable as ‘\( F \)’ on the table instead of smuggling in bundles of properties behind predicates, we should not negate or affirm either premise of the sorites argument. We should rather ask for Sir Sorites’ analysis of ‘bald’, or ‘child’, or whatever predicate is being used, such that Sir Sorites believes ‘\( Fa_1 \)’ and the inductive premise to be reasonable inferences from our use of such predicates in daily life.

There is something misleading about saying we understand what ‘tadpole’ means. This

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4Weiss presents a separate reply to Cargile’s objection [16, 268-271].
can only refer to our mastery the linguistic uses of the predicate for purposes of practical communication. Our successful linguistic use leaves obscure what property or properties ‘tadpole’ picks out. So we rather refuse to affirm or negate the second premise of sorites arguments until Sir Sorites lay their cards on the table, i.e., they explicate what bringing ‘bald’ into our logical vocabulary means. Perhaps there is a magic moment at which a tadpole ceases to be a tadpole, but we cannot say as of yet: the analysis is pending.

It may turn out that we end up with a property in our logical vocabulary such that a sorites argument goes through. What I propose here equally leaves open the eventual success of Sir Sorites in forcing us to reject some core principle like the law of bivalence. But there is no substitute for the honest toil; sliding from our rough-and-ready, practically-minded semantics for the predicate ‘bald’ to an item of our logical vocabulary is just Sir Sorites shirking the requisite work. We may rightly insist that Sir Sorites assuage our doubt that ‘bald’ picks out a property, and a unique one at that. Once Sir Sorites has done so, meaning, once he has provided an analysis that we would accept as specifying a unique and objective feature of the world, we have a true sorites paradox, and so also a genuine problem.

5.0. Throw Your Cap Over the Wall

I wish to address the worry that my challenge is unfair. What could count as a logical analysis of our successful linguistic use of ‘bald’, meaning, when would we know that a translation of the English predicate ‘bald’ into an item of our logical vocabulary, with a clearly defined semantic valuation function, was achieved? Absent some notion of when we could call such an analysis acceptable, we can be accused of a sort of willful ignorance as to what bald means. This is tantamount to rejecting any premise Sir Sorites might offer us.

5Dolev, in contrast, claims that the meaning of ‘bald’ makes any application of mathematical induction to baldness sorites paradoxes inappropriate in principle [4, 333-334]. I think Dolev is correct in a sense, but I avoid any question-begging against Sir Sorites: he may, if he wishes, strive to meet my definite challenge.
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Such a procedure is, of course, dialectically inappropriate, and a sort of surrender or quietism wherein we refuse to acknowledge that we understand even the ordinary predicate ‘bald’ insofar as we refuse to say that we mean anything by our words. And this denies the datum that our linguistic use of bald is successful - ‘bald’ must mean something, though we know not what at present. Hence, I want to indicate that analysis is possible [12, pp. 2, 111].

In the first place, analyses all must end in primitive predicates. Our logical vocabulary, if we choose to have one, must begin somewhere. A way for Sir Sorites to meet our challenge is to reduce the meaning of ‘bald’ to other terms that we feel more confident pick out objective features of our experience. If, for example, Sir Sorites suggested that ‘bald’ should mean ‘has fewer than one-thousand hairs above the eyebrow-line’, then we might accept this as an analysis of ‘bald’. The acceptability of the analysis will depend somewhat on what phrases or words you believe identify a unique and genuine property. But some of our English words must, if our logical language is to get off the ground (assuming English is our meta-language for our logic, that is). Assuming the chain of our defining terms does not continue infinitely, there is a ground-level set of semantical terms that we admit into our vocabulary, supposing we have one in our logical language, in which Sir Sorites can couch his analysis of ‘bald’. Sir Sorites need not define every word; he need only define his preferred soritical predicate in terms that his interlocutor agrees select a unique and genuine property. You will then truly have no escape from his soritical paradox and be forced to give up some dearly held premise.

Secondly, such analyses have already been offered in the past. Georg Cantor analyzed ‘infinity’ into the terms ‘aggregate’ and ‘element’ [2, 85]. Thus did Cantor arrive at novel and rigorous results concerning infinity. So also might Sir Sorites arrive at genuine soritical paradoxes, if he devoted an effort equal to Cantor’s. Another well-known example of analysis is Bertrand Russell’s paradox of the class which contains only those classes that do not contain themselves [11, 136]. This can be understood as a paradox that appeals to the accepted (at the time) analysis of ‘class’. And Guiseppe Peano analyzed arithmetic into

To take a prominent, non-mathematical example, consider the analysis of knowledge as a justified true belief. Edmund L. Gettier’s counterexamples to this analysis are prefaced on your accepting that what Gettier describes as Smith’s warrant for his own belief in one of the two thought experiments counts as justification [6, 122-123].

So these analyses can and have been given. Sir Sorites cannot object to the high burden upon his heaps of ‘heap’-style arguments if we in turn commit to the labor of analysis.
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References


